INTRODUCTION TO THE SPECIAL SERIES: MATHEMATICS AND LEARNING DISABILITIES

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Abstract. Increased attention is being paid to students who demonstrate difficulty in learning and applying mathematics concepts. The purpose of this special series was to address issues related to students and mathematics learning disabilities (LD). We identify Response to Intervention (RtI) as it relates to early mathematics instruction and a multi-tiered service delivery system. Further, because RtI has focused primarily on young children and the prevention of LD, we present information about older students who have been identified as having mathematics LD and provide strategies for helping them access the general education curriculum. Six papers on various mathematics topics, grade levels, and service delivery will be provided in this special series. Authors report findings on research efforts and offer implications for practice.

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Development and application of mathematical competence is a critical educational goal for all students, including those who have learning disabilities (LD). Yet, research on understanding mathematics disabilities and how they affect learning has lagged behind comparable work in reading disabilities. Likewise, when compared to the well-established research base in early reading difficulties, far less attention has been paid to early difficulties in mathematics and identification of mathematics disability (D. Bryant, Smith, & Bryant, 2008). Similarly, research on adolescent students with mathematics disabilities is limited at best.

Fortunately, in recent years, researchers have been paying increased attention to students who demonstrate challenges in learning and applying mathematics skills and concepts (D. Bryant, 2005; Chard et al., 2005; Fuchs & Fuchs, 2005; Gersten, Jordan, & Flojo, 2005; Jitendra et al., 2005; Jordan, Hanich, & Kaplan, 2003; Kroesbergen & Van Luit, 2003; Maccini & Gagnon, 2006; Miller, & Hudson, 2007; Montague, 2006). This special series on mathematics and LD is intended to add to this growing research base.

The specific purpose of this special series is to address two primary issues related to students and mathematics learning disabilities. First, at a time when Response to Intervention (RtI) is the focus of considerable attention and discussion in the educational community, we seek to identify key RtI issues as they relate to early mathematics instruction, particularly as they pertain to a multi-tiered service delivery system. Second, because RtI has focused primarily on young children and prevention of LD, we present information about older students who have been identified as having mathematics LD and provide strategies they can use to access the general education curriculum.

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In this introduction, we (a) describe characteristics of students who have difficulties in mathematics, (b) overview the purposes of RtI and the multi-tiered service delivery system, (c) discuss briefly elementary and secondary math instruction, and (d) introduce the articles that constitute the special series and the authors who contributed to the series.

**Characteristics of Students with Mathematics Difficulties**

Mathematical difficulties are persistent and evident from the early elementary grades through secondary levels (Garnett, 1987). With respect to the early grades, Gersten et al. (2005) examined several kindergarten through grade 2 studies that compared students who exhibited mathematical difficulties to their typically achieving peers. Learning problems were found in arithmetic combinations (i.e., basic facts), counting strategies (e.g., counting all, counting on), and number sense (e.g., basic counting techniques, understanding of size of number, number relationships). Based on their findings, Gersten and his colleagues suggested that, over a period of time, limited mastery of arithmetic combinations (basic facts) was a “hallmark” of mathematics difficulties.

In a study on mathematics difficulties with students in grades 2 through 12, D. Bryant et al. (2000) identified 29 specific mathematics behaviors associated with math LD based on a search of the research and theoretical literature (see Table 1). They then asked a group of LD teachers to rate the frequency with which students exhibited the identified mathematical skills. A rank ordering of responses showed that certain mathematics problems were troublesome for students with mathematics weaknesses across ages. Not surprisingly, word problems were ranked as most problematic for students with learning disabilities and math weaknesses. Table 1 shows the ranking of the mathematical difficulties, in decreasing order of frequency, exhibited by students with LD and math weaknesses.

D. Bryant et al. (2008) further surveyed the literature and identified specific behaviors associated with math calculation and problem solving demonstrated across the grades. Students with calculations difficulties may demonstrate problems with some or most of the following skills:

- Identifying the meaning of signs (e.g., +, -, x, <, =, >, %, $\Sigma$)
- Remembering answers to basic arithmetic combinations (e.g., $8 + 9 = \ ?$, $7 \times 7 = \ ?$)
- Using effective counting strategies to calculate answers to arithmetic problems.
- Understanding the commutative property (e.g., $5 + 3 = 8$ and $3 + 5 = 8$)
- Solving multi-digit calculations that require regrouping
- Misaligning numbers
- Ignoring decimal points

Word problem-solving difficulties may be observed in any of the following skills:

- Reading the problem
- Understanding the meaning of the sentences
- Understanding what the problem is asking
- Identifying extraneous information that is not required for solving the problem
- Developing and implementing a plan for solving the problem
- Solving multiple steps in advanced word problems
- Using the correct calculations to solve problems

An examination of the behaviors cited across the studies demonstrates that difficulties are broad-based and exhibited across a wide array of mathematics content. While individual students with mathematics LD may exhibit only one or two of the listed behaviors, they are more likely to exhibit pervasive difficulties (D. Bryant et al., 2008). Given the mathematics difficulties demonstrated by students with LD, prevention and intervention are critical components to include as part of instructional delivery (Fuchs & Fuchs, 2001).

**Overview of the Purposes of RtI and the Tiered Service Delivery System**

For years, LD diagnoses were based primarily on a discrepancy between intelligence test scores and achievement test scores. The Individuals with Disabilities Education Improvement Act – IDEA 2004 – allows for either the IQ-achievement discrepancy model or the newly conceived RtI approach for identifying learning disabilities. With the RtI approach, once students have been identified as having low achievement and learning at a rate below that of the their nondisabled peers, they may be identified as having learning disabilities (provided they meet exclusionary clause criteria) (Vaughn & Fuchs, 2003).

Vaughn (2002) described one RtI approach as a prevention and intervention model for struggling readers in the early grades. The 3-Tier Reading Model provides a framework for a data-driven delivery of differentiated instruction for all students (Vaughn Gross Center for Reading and Language Arts [VGCRRA], 2005) that can be generalized to mathematics or any other academic area. Tier 1 consists of evidence-based core instruction for all students, including struggling students receiving concurrent services in Tier 2 and Tier 3 (D. Bryant & Bryant, 2007). Tier 2 provides supplemental instruction and ongoing progress monitoring to students who have been identified as achieving below their average- and above-average-performing peers. The classroom teacher
Table 1

**Ranked Mathematics Difficulties Exhibited by Students with Learning Disabilities and Math Weaknesses.**

- Has difficulty with word problems
- Has difficulty with multi-step problems
- Has difficulty with the language of math
- Fails to verify answers and settles for first answer
- Cannot recall number facts automatically
- Takes a long time to complete calculations
- Makes “borrowing” (i.e., regrouping, renaming) errors
- Counts on fingers
- Reaches “unreasonable” answers
- Calculates poorly when the order of digit presentation is altered
- Orders and spaces numbers inaccurately in multiplication and division
- Misaligns vertical numbers in columns
- Disregards decimals
- Fails to carry (i.e., regroup) numbers when appropriate
- Fails to read accurately the correct value of multidigit numbers because of their order and spacing
- Misplaces digits in multidigit numbers
- Misaligns horizontal numbers in large numbers
- Skips rows or columns when calculating (i.e., loses his or her place)
- Makes errors when reading Arabic numbers aloud
- Experiences difficulties in the spatial arrangement of numbers
- Reverses numbers in problems
- Does not remember number words or digits
- Writes numbers illegibly
- Starts the calculation from the wrong place
- Cannot copy numbers accurately
- Exhibits left-right disorientation of numbers
- Omits digits on left or right side of a number
- Does not recognize operator signs (e.g., +, -)

or an intervention specialist may provide Tier 2 instruction. Tier 3 is reserved for students who have not responded to Tier 1 instruction and supplemental Tier 2 instruction and who require additional intensive intervention and continuous progress monitoring. Tier 3 intervention may be delivered by a special education teacher or an academic specialist.

Fuchs and Fuchs (2001) identified a similar framework for prevention and intervention of mathematics difficulties, which consists of primary, secondary, and tertiary instruction. Primary prevention focuses on universal design, or instruction that benefits all students, including those with learning problems. Secondary prevention involves instructional adaptations that are feasible to implement, nondisruptive to the targeted child, and nonintrusive for the other students in the class. Finally, tertiary prevention is intensive and individualized, involving special services that are often provided by the special education teacher.

Some researchers and practitioners have developed systems that involve four or even five tiers (Dickson & Bursuck, 1999; Grizel, n.d.). But all systems have similar goals: to (a) provide instruction to students in growing levels of intensity, (b) reduce the numbers of students receiving instruction in the succeeding tiers, and (c) identify a group of students who may qualify as having LD.

Although RtI is often discussed concerning its contribution to LD identification, the majority of the current research and writings focus on RtI as a prevention service delivery model. How RtI can be operationalized successfully and provide for a means to identify LD remains to be validated. Within the context of RtI, instructional approaches for teaching elementary and secondary mathematics are employed. We offer a glimpse of these approaches to frame this series.

**Elementary and Secondary Mathematics Instruction**

The initial standards-based reform efforts of the late 1980s continue to influence core mathematics instruction at the elementary and secondary levels. As Jitendra and her colleagues (2005) noted, “An important implication of the standards-based reforms is that complex, higher order thinking and problem solving have become integral to the concept of what constitutes learning in general education classrooms” (p. 320). Certainly, all students should engage in meaningful instructional activities that promote higher order thinking, reasoning, and problem solving.

In response to the standards developed by the National Council of Teachers of Mathematics (NCTM), core mathematics instruction in most of today’s elementary and secondary classrooms focuses on an inquiry-based, or discovery, approach to solving mathematical problems (Baxter, Woodward, & Olson, 2001). Thus, teachers spend considerable time and effort during mathematics instruction ensuring that students interact with their peers to develop solution strategies for problems that focus on a range of mathematical concepts (e.g., number sense, statistics and probability, measurement, geometry). Students share their mathematical reasoning and solutions during whole-class instruction and small-group breakout sessions (Baxter et al., 2001). Thus, core mathematics instruction may embrace activities that allow students to demonstrate their conceptual understanding and reasoning rather than their abilities in memorization and rote learning (Jitendra et al., 2005; NCTM, 2000).

Woodward (2004), however, noted that findings from naturalistic research in reform-based mathematics classrooms suggest that discovery, in and of itself, may be insufficient for some students with mathematics difficulties; these students require mediated instruction to enable them to be actively engaged in the learning process. Such instruction may be characterized by teacher questioning to guide learning more explicitly and by systematic, strategic instruction to teach concepts and skills that students may lack. For example, findings from Swanson, Hoskyn, and Lee’s (1999) meta-analysis of academic treatment outcomes for students with LD confirmed the benefits of employing procedures from explicit, systematic, and strategic instruction in a combined instructional approach.

Explicit instruction focuses on the teaching of sub-skills (e.g., division facts, multi-digit dividend divided by a one-digit divisor with no remainder) that are targeted for instruction, modeling, practice, error correction, and progress monitoring. Strategic instruction focuses on using a combination of procedural rules, metacognitive (e.g., self-regulatory) cues, memory retention and retrieval techniques, and mnemonics. Strategic instruction provides a rationale for learning a particular strategy and teaches specific steps designed to activate cognitive and metacognitive processes.

These instructional procedures may take place in small-group pullout sessions as part of Tier 2 or Tier 3 instruction or as part of typical instruction that takes place in the general education classroom with or without special education support. Small-group instruction is particularly effective in providing explicit and strategic instruction to address the wide range of student responses to core mathematics instruction at all grade levels. That is, in small groups, students have more opportunities to practice what they know and receive immediate feedback from the teacher and other students (Vaughn, Hughes, Moody, & Elbaum, 2001). Additionally, small-group instruction is a distinguishing...
characteristic of secondary and tertiary interventions in RtI (VGCRLA, 2005). Thus, as teachers become increasingly accountable for employing RtI procedures (e.g., evidence based instructional practices, progress monitoring of student performance, tiered instruction) in mathematics instruction, they must be provided the means for doing so. In this special series, recommendations for mathematics instruction and progress monitoring are described across the grade levels.

**Introduction to the Special Series' Articles and Authors**

This is the first of a two-part special series that includes articles representing different grade levels (kindergarten, elementary, secondary) and mathematics topics with a focus on students who have been identified as having mathematics difficulties or disabilities. Additionally, the series includes articles that examine practices for the different tiers (core or primary, secondary, tertiary) of instruction to prevent mathematics difficulties and to provide more intensive intervention to those students most in need of this assistance. To that end, we introduce the authors and topics of Part 1 and Part 2 of the special series in mathematics and learning disabilities, being mindful that in the primary grades typically learning disabilities are not identified.

**Part 1 of the special series.** Part 1 consists of three articles in addition to this introduction. David Chard and his colleagues (Scott Baker, Ben Clarke, Kathleen Jungjohann, Karen Davis, and Keith Smolkowski) discuss what kindergarten Tier 1 (or core) instruction should look like and describe their efforts to create a core kindergarten mathematics program that would help all young children learn key concepts and skills. Chard et al. present research findings on their early mathematics program, demonstrating the potential value of their curriculum. In a mathematics prevention model, kindergarten is a critical year for students to develop or enhance their number sense and numeration skills. Chard and his co-authors provide an excellent description of key features that should be present in the early core mathematics curriculum and provide a template for others to follow as they validate instructional programs.

Next, along with members (or former members) of our mathematics research team (Caroline Kethley, Sun A Kim, Cathy Pool, and You-Jin Seo), we report the findings of our basal evaluation study. Eleven critical features of effective instruction, including content and procedures, for teaching mathematics skills to all students (struggling students in particular) were examined in four basals. The critical features, drawn from the research and theoretical literature on early mathematics instruction, provide readers with a means to evaluate how well basal textbooks might meet the needs of early struggling learners. Although the research focuses on kindergarten and grades 1 and 2, we argue that the features discussed may appropriately be examined in mathematics texts at all grade levels.

The final article in this issue was contributed by Marjorie Montague. Montague presents her research on self-regulation and how it affects the story-problem performance of middle school students who have learning disabilities. Specifically, she discusses the importance of self-regulation, describes key principles associated with strategy instruction, reviews research that she and others have conducted on self-regulation and mathematics problem solving, and provides guidelines for implementing cognitive strategy instruction.

**Part 2 of the special series.** In the next issue, we and other SERP-M (Special Education Research Project: Mathematics) colleagues (Russel Gersten, Nancy Scammacca, Catherine Funk, Amanda Winter, and Minyi Shih) report on an early mathematics intervention program that we developed and have been field testing and researching for the last two years. Incorporating explicit, strategic instruction, the program's lessons have proven to be effective in helping students develop critical early math skills. We review a segment of the project that deals with helping first graders in Tier 2 improve their number and operations skills. The progress monitoring instruments developed and validated for the project, key elements of effective instruction incorporated into the intervention program, and results of the yearlong intervention are presented. The practical significance of an early mathematics prevention and intervention model is also described.

In the second article, Anne Foegen reports on her work in designing progress-monitoring measures for secondary-level algebra content. Progress monitoring is a key feature of effective instruction and plays an important role in ensuring that interventions are having the desired effect. Because most available progress-monitoring measures focus on early math skills, Foegen's work is pertinent to researchers and practitioners who work with older students who have learning disabilities. Foegen discusses algebra instruction and presents her research on progress-monitoring devices she has developed and validated. She concludes by discussing what she sees as the future for algebra instruction and assessment with students who have LD.

Finally, Lynn Fuchs and her colleagues (Douglas Fuchs, Sarah Powell, Pamela Seethaler, Paul Cirino, and Jack Fletcher) wrap up our series with a discussion of their work with students receiving tertiary instruction. Most of the research and writings in prevention have focused on core instruction and secondary interventions. But what is being done for students who struggle
while receiving secondary interventions and require more intensive, tertiary instruction? L. Fuchs and her colleagues provide answers to this question by presenting the principles of effective practice used in intensive intervention, describing their validated intervention practices for third-grade students, and discussing how progress monitoring and intensive instruction go hand in hand across grade levels.

We are grateful to our colleagues who have offered important contributions to a growing research base on mathematics service delivery, instruction, and progress monitoring for older students who have LD and prevention efforts for young struggling students. We hope that readers find the articles in this special series helpful as they conduct research or provide mathematics instruction to students who struggle with mathematics and have mathematics learning disabilities.

REFERENCES


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